Proximity effect can induce the energy gap rather than superconducting pair potential

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Abstract

For an s-wave superconductor/semiconductor/ferromagnetic-insulator structure, the proximity effect can induce the energy gap in the semiconductor rather than the superconducting pair potential of its microscope Hamiltonian. As a result, it is questionable to realize topological superconducting states in that structure.

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The experimental realization of Majorana fermions has triggered an avalanche of research activity, because such excitations satisfy non-Abelian statistics which form a centerpiece in recent proposals for topological quantum computations. In a recent Letter, Sau *et al.*[1] proposed that a topological superconducting phase supporting Majorana fermions can be realized using a semiconductor with Rashba spin-orbit coupling, sandwiched between an s-wave superconductor (S) and a ferromagnetic insulator. The basic idea is that the ferromagnetic insulator produces a Zeeman field V_z perpendicular to the semiconductor, which separates the two spin-orbit-split bands by a finite gap. For $\mid \mu \mid < \mid V_z \mid$, i.e., the Fermi level μ lies inside this gap, only a single band crosses the Fermi level, which is analogous to the surface state of a strong topological insulator (TI).[2] There will appear non-abelian topological order if s-wave superconductivity is induced in the semiconductor by the proximity effect. In this system, the superconducting pair potential Δ is a key ingredient of driving the semiconductor into a topological superconducting state. However, we argue here that Δ assumed in the semiconductor [Eq. (2) of Ref. 1] is questionable, even though the proximity effect can induce the superconducting order parameter and the energy gap (not the superconducting energy gap) there.

It is of great importance to distinguish pair potential Δ appeared in the microscopic Hamiltonian from the energy gap and superconducting order parameter. The pair potential stands for the number of Cooper pairs in the condensate, and it determines the basic features of superconductivity such as the Meissner effect and vanishing electric resistance. For an S/normal-metal (N) bilayer, both superconducting order parameter $F(z)$ and pair potential $\Delta(z)$ always exist on the S side, which satisfy a simple relation of $\Delta(z) = \lambda^* F(z)$ with λ^* the effective pair interaction.[3] On the N side, $F(z)$ can be induced by the proximity effect, but $\Delta(z)$ determined from the self-consistency equation equals zero because λ^* is supposed to be vanishing. The superconducting order parameter in the N indicates the quantum coherence of a pair of electrons without condensate. As regards the energy gap, it is not the mark of superconductivity. In an S, the superconducting energy gap is the minimal binding energy of the Cooper pair in the condensate, and the gapless superconductivity may appear due to magnetic impurities or the inverse proximity effect. In an N contact to an S, the proximity-effect-induced energy gap is not a superconducting energy gap, but a pseudo energy gap arising from the coherence of electron pairs without condensate. Another example of the pseudo energy gap is the normal state of the high-T_c superconductors, where there is a

finite energy gap but no superconductivity. As a result, without the BCS pair potential, the proximity-effect-induced energy gap ought not to be viewed as a basis of assuming a finite pair potential in the microscope Hamiltonian.

The pair potential cannot be assumed artificially, and it must be self-consistently determined as long as it appears in the microscope Hamiltonian and BdG equation. If both Δ and V_z are assumed to appear simultaneously in the microscope Hamiltonian, they must be related to each other. For such an assumption, it is necessary to show that the coexisting state for Δ and V_z has the lowest thermodynamic potential. In the absence of the spin-orbit coupling, from the lowest thermodynamic potential principle, it has been shown that the s-wave superconducting ground state can be realized only for $0.707\Delta > V_z$, which is the Clogston criterion at zero temperature.[4] However, the existence of non-Abelian Majorana fermions requires the condition of $(\mu^2 + \Delta^2) < V_z^2$, i.e., at least $\Delta < V_z$. Although the presence of the spin-orbital interaction complicates the question, the lowest thermodynamic potential principle cannot be violated.

In summary, we argue that the pair potential in the microscope Hamiltonian of the semiconductor cannot be induced by the proximity effect. As a result, it is questionable to realize the topological superconducting state in the s-wave S/semiconductor/ferromagnetic-insulator structure.[1]

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